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2668. Proposed by B. F. FINKEL, Drury College.

Show that

$$v = \frac{2}{9}\frac{ga^2\sigma}{\eta},$$

where a is the radius of a droplet, σ its density, η the viscosity of the air and v the velocity under gravity g. *Stokes's law.*

SOLUTION BY ALBERT R. NAUER, St. Louis, Missouri.

Stokes finds analytically a formula determining the resistance of a fluid medium to the movement of a sphere through it,

$$F = \tfrac{4}{3}\pi\rho an\sqrt{-1}[af_1'(a) + 2f_2(a)]e^{\sqrt{-1}nt}, \tag{I}$$

Stokes's *Mathematical and Physical Papers*, Vol. III, p. 33. From a previous development (p. 28) is given

$$f_1''(r) - \frac{2}{r^2}f_1(r) = 0, \qquad \text{and} \qquad f_2''(r) - \frac{2}{r^2}f_2(r) - m^2f_2(r) = 0. \tag{II}$$

Then the equations of condition become on putting $f(r)$ for $f_1(r) + f_2(r)$, and $r = a$,

$$f'(a) = ac, \qquad f(a) = \tfrac{1}{2}a^2c, \tag{1}$$

$$f'(b) = 0, \qquad f(b) = 0. \tag{2}$$

The integration of the differential equations (II) has no difficulties, the first one is of a well-known form and the second one is a Riccati equation solvable in finite terms.

The integrals are

$$f_1(r) = \frac{A}{r} + Br^2 \qquad \text{and} \qquad f_2(r) = Ce^{-mr}\left(1 + \frac{1}{mr}\right) + De^{mr}\left(1 + \frac{1}{mr}\right).$$

Here $D = 0$, since otherwise the velocity would be infinite at an infinite distance; $B = 0$, otherwise the velocity would be finite when $r = \infty$.

From Eq. (1), we get

$$A = \tfrac{1}{2}a^3c - aCe^{-ma}\left(1 + \frac{1}{ma}\right).$$

Putting $b = \infty$, we get from Eq. (2),

$$\frac{d}{da}\{\text{Equation (1)}\} = 0 = \frac{3a^2c}{2} - aC(1 - m - 1)e^{-ma}, \qquad C = \frac{3ac}{2m}e^{ma},$$

and then

$$A = \tfrac{1}{2}a^3c + \frac{3a^2c}{2m}\left(1 + \frac{1}{ma}\right).$$

Substituting in (I),

$$F = -\tfrac{2}{3}\pi a^3cn\sqrt{-1}\left(1 + \frac{9}{ma} + \frac{9}{m^2a^2}\right)e^{\sqrt{-1}nt}. \tag{III}$$

Substituting for m its value, which is the particular square root of $(n\sqrt{-1})/\mu'$ which has its real part positive, and write V for $ce^{\sqrt{-1}nt}$. Then

$$F = -6\pi\mu'\rho aV, \tag{IV}$$

where $\mu' = \mu/\rho$, called by Stokes, the *index of friction* and ρ is the density of the fluid. Whenever the motion is so slow that the part of the resistance which depends on the square of the velocity may be neglected, we have, supposing V to be the terminal velocity;

$$-F = \tfrac{4}{3}\pi ga^3(\sigma - \rho),$$

where g is the force of gravity, and σ, which is supposed to be greater than ρ, the density and a the radius of the sphere.

Hence,

$$6\pi\mu'\rho a V = \tfrac{4}{3}\pi g a^3(\sigma - \rho),$$

or

$$V = \frac{2}{9}\frac{g}{\mu'}\left(\frac{\sigma}{\rho} - 1\right)a^2 = \frac{2}{9}\frac{ga^2\sigma}{\eta},$$

by placing

$$\frac{\sigma - \rho}{\mu\rho} = \frac{\sigma}{\eta}.$$

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas.

DISCUSSIONS.

CONCERNING TWO FIFTH-POWER PROBLEMS IN DIOPHANTINE ANALYSIS.

By CYRUS B. HALDEMAN, Ross, Butler County, Ohio.

I. *To resolve*

$$p^5 + q^5 + p^5 + s^5 = t^5 + u^5 + v^5 \tag{1}$$

where p, q, r, s, t, u, and v are rational.

Solution. Let $p = 2x$, $q = y - x$, $r = 2a - b^2 - x$, $s = 2a + b^2 + x$, $t = x + y$, $u = 2a - b^2 + x$ and $v = 2a + b^2 - x$.

Substituting in (1) we get

$$\begin{aligned}(2x)^5 + (y - x)^5 + (2a - b^2 - x)^5 + (2a + b^2 + x)^5&\\ = (x + y)^5 + (2a - b^2 + x)^5 + (2a + b^2 - x).^5&\end{aligned} \tag{2}$$

We may write (2):

$$\begin{aligned}(x + y)^5 + (x - y)^5 + [(2a + b^2) - x]^5 - [(2a + b^2) + x]^5&\\ + [(2a - b^2) + x]^5 - [(2a - b^2) - x]^5 - (2x)^5 = 0.&\end{aligned} \tag{3}$$

Expanding, regarding each binomial in parenthesis as a single quantity, and adding, we have from (3)

$$\begin{aligned}2x^5 + 20x^3y^2 + 10xy^4 - 2x^5 - 20x^3(2a + b^2)^2 - 10x(2a + b^2)^4&\\ + 2x^5 + 20x^3(2a - b^2)^2 + 10x(2a - b^2)^4 - 32x^5 = 0.&\end{aligned} \tag{4}$$

Equation (4) reduces to

$$2x^2y^2 + y^4 - 16ab^2x^2 - 64a^3b^2 - 16ab^6 - 3x^4 = 0.$$

Solving for y^2, we get

$$y^2 + x^2 = \sqrt{4x^4 + 16ab^2x^2 + 64a^3b^2 + 16ab^6} = 2x^2 + 4a(b^2 + d^2), \text{ say.}$$